# Multi-scenarios Multi-objective Predictive Controller For Lateral Control Of Autonomous Vehicles

**Joyce SUDI** – PhD Student

with Rodolfo ORJUELA, Michel BASSET and Imane MAHTOUT

Stellantis – IRIMAS Institute - UR 7499

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# Agenda



# Context

- Fully **automated driving** - Autonomous vehicles in urban areas and on highways
- Passenger comfort and greater acceptance of automated driving
- Safety for passengers and vulnerable road users.
    - 40% of fatalities in urban areas in Europe are vulnerable road users

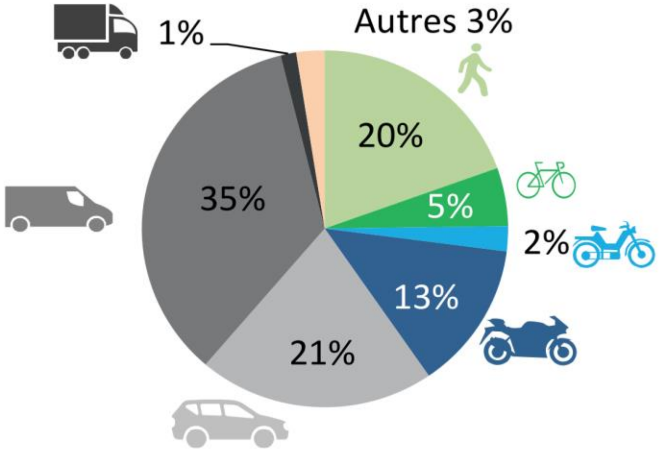



Fig. Fatalities in accidents involving a passenger car by mode of transport in 2023 (Source: ONISR[1])

[1] Observatoire national interministériel de la sécurité routière (ONISR, French road safety observatory)

# System design criteria

- (Req1.) Tracking quality:

  Minimum tracking error, desired settling-time in step response, etc.

- (Req2.) Ride comfort:

  Minimum acceleration, minimum lateral and steering jerks.

- (Req3.) Safety:

  System limitations including physical constraints of actuators and driving environment requirements like maintaining safe distances.

- (Req4.) Robustness:

  Consistent performance under a variety of conditions (e.g. external disturbances, model mismatch, environmental noise, etc.).

The outlined requirements (Req1., Req2., Req3., Req4.) lead to a **multi-objective problem** between performance, safety and comfort, with potentially conflicting objectives.

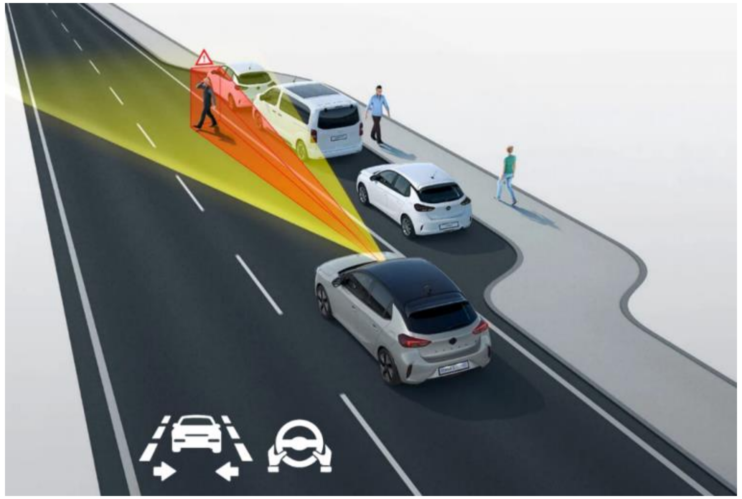
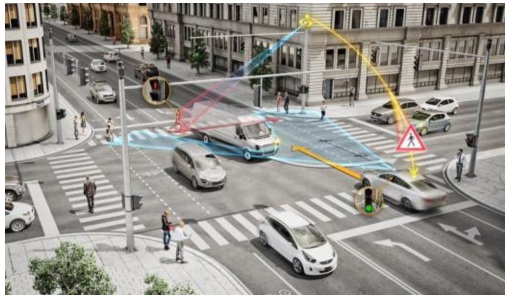


Fig. Lane centering assistance (LCA). Pedestrian crossing leading to a conflict situation.

# Challenges

- The use of Model Predictive Control (MPC) in contexts where operational conditions may dynamically change, such as in scenarios requiring on-the-fly or online adjustments.
- The controller needs to adapt its operational mode in response to external variables that are subject to online, time-varying changes.
- Guarantee classical system-theoretic properties during dynamic operations, including recursive feasibility, constraint satisfaction, stability and consistent performance for general nonlinear systems.

### Urban traffic



Low speed and high steering control:

Traffic congestion, presence of vulnerable road users, presence of traffic circles, etc.

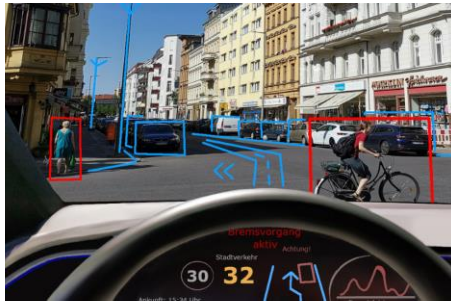
### Highway traffic


High speed and low steering control:

Obstacle avoidance, lane changes, overtaking

### Uncertainties and variation


Unknown and Changing Environments




Unpredictable Events

Credits: Etech (https://etech.iec.ch)

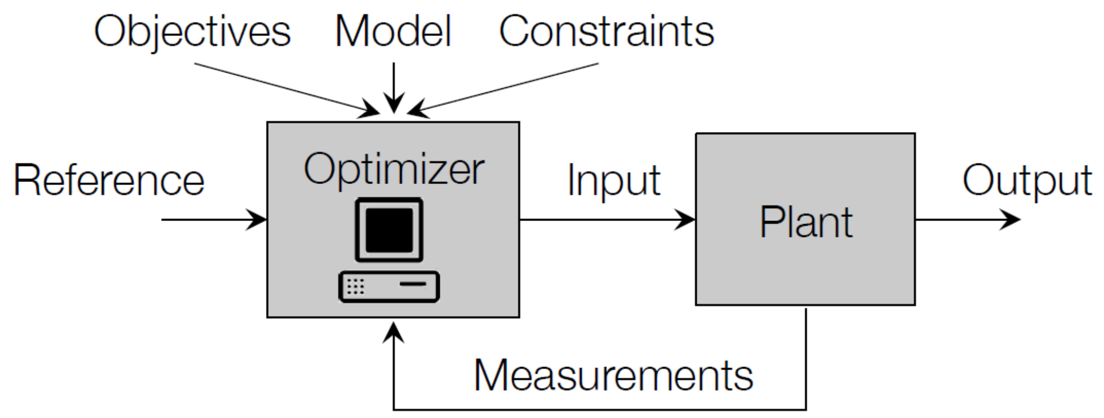
# Objectives

- The adjustment of the MPC's structure, including its prediction horizon, constraints, costs, and decision variables, is required to align with the current driving situation in a multi-scenario context.
- Development of Multi-scenarios multi-objective driving algorithms to cover a greater number of driving scenarios
- Incorporating safety measures and practical constraints (acceleration, lateral & steering jerks, etc.), including regulations (speed limit, safety distance) and system limits (actuator limits)

# Related works


- Mustaki S., Chevrel P., Yagoubi M., and Fauvel F. (June 2018). Efficient Multi-Objective and Multi-Scenarios Control Synthesis Methodology for Designing a Car Lane Centering Assistance System. “In 2018 European Control Conference”. Limassol, Cyprus.
- Atoui, H., Sename, O., Milanés, V., and Martinez, J. (2022). LPV-Based Autonomous Vehicle Lateral Controllers : A Comparative Analysis. IEEE Transactions on Intelligent Transportation Systems, 23(8): 13570–13581.
- Pereira, G. C., Lima, P. F., Wahlberg, B., Pettersson, H., and Mårtensson, J., (December 2018). Linear Time-Varying Robust Model Predictive Control for Discrete-Time Nonlinear Systems. “In 2018 IEEE Conference on Decision and Control”. Miami Beach, FL, USA.
- Köhler, J., Müller, M. A., & Allgöwer, F. (2024). Analysis and design of model predictive control frameworks for dynamic operation—An overview. Annual Reviews in Control, 57, 100929.

# Optimization in the Loop: The Receding-horizon principle[1]



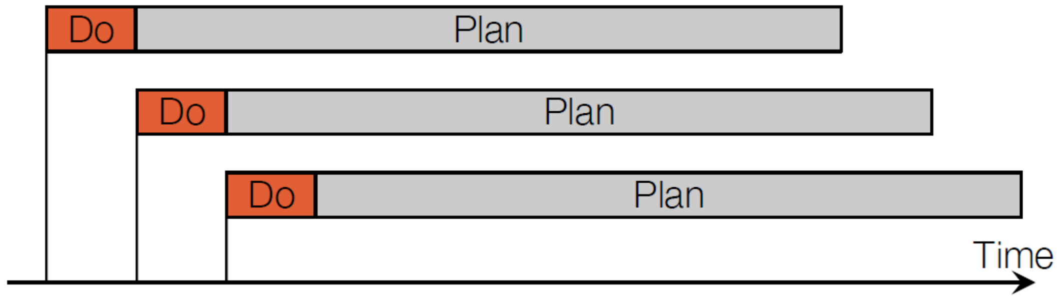




Fig. The Receding-horizon principle

## Reasons to Use Predictive Control

1. Constraints drive performance
2. Strongly nonlinear system dynamics
3. Complex objectives
4. Some future knowledge

> If we are to better the future, we must disturb the present
> — Catherine Booth

[1]Colin Jones, Lecture Notes on Model Predictive Control (EPFL ME-425), 2021.

# Vehicle modelling

## Lateral dynamic model in terms of error with respect to road[1]

Let denote the state and control vectors, respectively, as:

$$x = [\ e_y,\ \dot{e}_y,\ e_\psi,\ \dot{e}_\psi\ ]^T, \quad u = \delta_f \tag{4}$$

State space representation of the vehicle model:

$$\dot{x} = Ax + Bu \tag{5}$$

being:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{a}{mv_x} & \frac{b}{m} & -v_x - \frac{b}{mv_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{b}{I_z v_x} & \frac{b}{I_z} & -\frac{c}{I_z v_x} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{2C_{\alpha_f}}{m} \\ 0 \\ \frac{2l_f C_{\alpha_f}}{I_z} \end{bmatrix} \tag{6}$$

$$a = 2\left(C_{\alpha_f} + C_{\alpha_r}\right), \quad b = 2\left(C_{\alpha_f} l_f - C_{\alpha_r} l_r\right)$$
$$c = 2\left(C_{\alpha_f} l_f^2 + C_{\alpha_r} l_r^2\right)$$

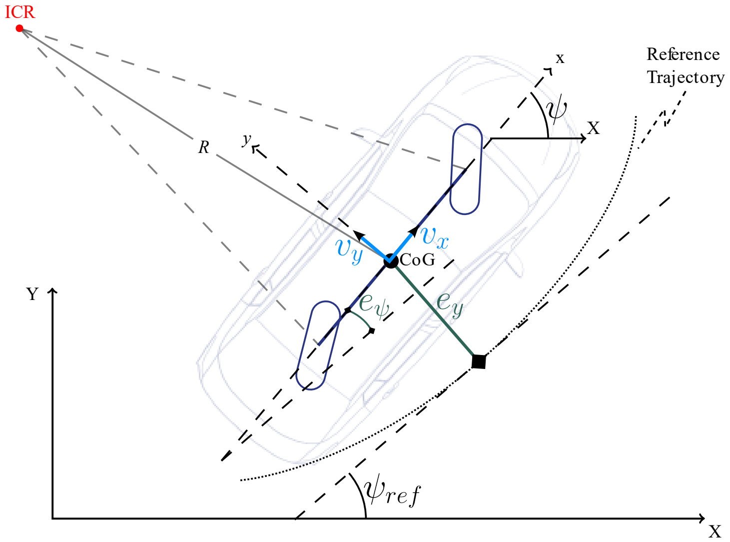



Fig. bicycle model with both the global and road-aligned coordinate frames

With :

- $e_y$ , the lateral position error with respect to road
- $e_\psi$ , the yaw angle error with respect to road

# LPV model representation[2]

After choosing $\rho = v_x$ as the varying parameter, the model (5) can be written in a LPV form:

$$\dot{x} = A(\rho)x + B(\rho)u \tag{7}$$

[1] Rajesh Rajamani. Vehicle Dynamics and Control. Springer Science & Business Media, December 2011.
[2] Atoui, H., et al. (2022). LPV-Based Autonomous Vehicle Lateral Controllers: A Comparative Analysis. IEEE Transactions on Intelligent Transportation Systems, 23(8), 13570-13581.

# States Constraints

## Environmental envelopes (driving corridor) (Req3.)

Collision avoidance is enforced using a driving corridor[1]:
In a road-aligned frame, these bounds can be enforced on the vehicle's lateral error as follows:

$$e_{y,min,i} \leq e_{y,i} \leq e_{y,max,i}\ , \quad i = 1, \ldots, N \tag{8}$$

Fig. Driving corridor

## Vehicle handling stability (Req3.)

Vehicle sideslip angle[2]

$$\beta = -arctan(\frac{v_y}{|v_x|}) = \frac{1}{v_x}\dot{e}_y - e_\psi \tag{9}$$

The used envelope in the controller formulation is given by[3]:

$$\beta_{max} = 10^\circ - 7^\circ \frac{v_x^2}{40(m/s)^2} \tag{10}$$

[1] Erlien, S.M., Fujita, S., and Gerdes, J.C. (2013). Safe Driving Envelopes for Shared Control of Ground Vehicles. IFAC Proceedings Volumes, 46(21), 831-836.
[2] Rajesh Rajamani. Vehicle Dynamics and Control. Springer Science & Business Media, December 2011.
[3] Uwe Kiencke and Lars Nielsen. Automotive Control Systems: For Engine, Driveline, and Vehicle. Springer Science & Business Media, April 2005

# Input Constraints

## Driving comfort and operational safety

1. Physical limits of the Actuators (hard-constraint) (Req3.)

$$\begin{aligned} \delta_{min} &\leq \delta \leq \delta_{max} \\ \dot{\delta}_{min} &\leq \dot{\delta} \leq \dot{\delta}_{max} \\ \ddot{\delta}_{min} &\leq \ddot{\delta} \leq \ddot{\delta}_{max} \end{aligned} \tag{11}$$

2. Ride comfort (soft-constraint): to limit the maximum permitted lateral acceleration[1] (Req2.)

$$\delta_{max}(v_x) = \frac{l}{v_x^2} a_{y,base} + \mathcal{E}_{margin} \tag{12}$$

With :

- $a_{y,base}$ , the maximum admissible lateral acceleration
- $l$ , the distance between the front and rear axles.

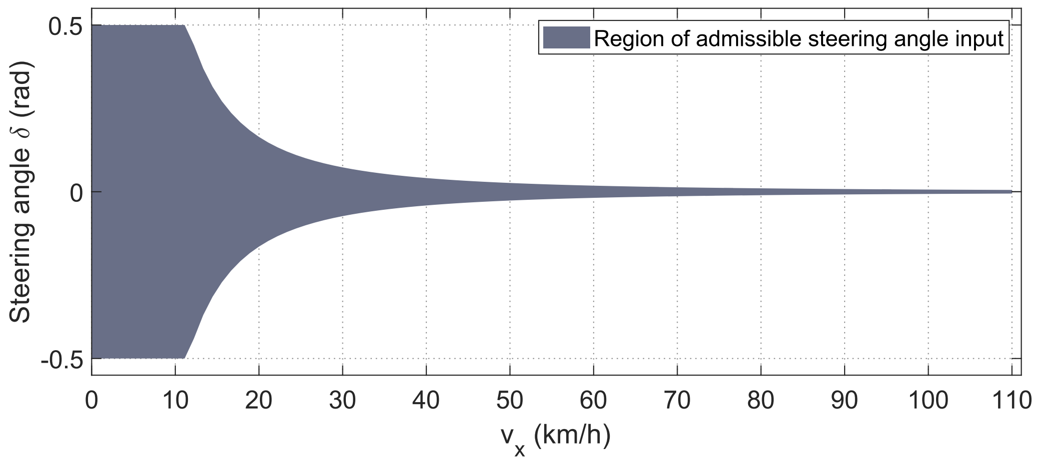



Fig. Steering angle $\delta$ constraints.

[1] E. Kim et al. Model predictive control strategy for smooth path tracking of autonomous vehicles with steering actuator dynamics. International Journal of Automotive Technology, 15(7):1155–1164, 2014.

## Optimal Control Problem Formulation

For the design of LPV-MPC, a discrete model is required.
The model (7) is discretized using the Forward-Euler method with sampling time Ts = 0.02 s:

$$x(k+1) = A_d(\rho)x(k) + B_d(\rho)u(k) \tag{13}$$

The environment envelopes (5) and vehicle handling stability (7) can be represented compactly in the Optimal Control Problem (OCP) as:

$$\mathbb{X}^k = \left\{x_i | F^k x_i \leq f^k\right\}, \quad i = 1 \dots N \tag{14}$$

The physical limitations of the steering system (8) and driving comfort constraints (9) can be gathered into:

$$\mathcal{U}^k = \left\{u_i | G^k u_i \leq g^k\right\}, \quad i = 1 \dots N-1 \tag{15}$$

# Optimal Control Problem Formulation

The proposed multi-objective LPV-MPC problem is written as:

$$
\begin{aligned}
\min_{u,\mathcal{E}_{x_n},\mathcal{E}_{comf}} \quad & J_{track} + J_{comf} + J_{prio} \\
s.t. \quad & x_{i+1/k} = A_d(\rho(k))x_{i/k} + B_d(\rho(k))u_{i/k} \\
& x_{0/k} = x_k \\
& u_{i/k} \in \mathcal{U}^k \\
& x_{i/k} \in \mathbb{X}^k, \quad x_{N/k} \in \mathcal{X}_f^k \\
& \mathcal{E}_{x_n/k} \geq 0, \quad \mathcal{E}_{comf,i/k} \geq 0, \quad i = 0 \dots N-1
\end{aligned}
\tag{16}
$$

being:

$$
\begin{aligned}
J_{track} &= \sum_{i=0}^{N-1} x_{i/k}^T Q x_{i/k} + u_{i/k}^T R_0 u_{i/k} + x_{N/k}^T Q_f x_{N/k} \\
J_{comf} &= \sum_{i=0}^{N-1} \dot{u}_{i/k}^T R_1 \dot{u}_{i/k} + \ddot{u}_{i/k}^T R_2 \ddot{u}_{i/k} \\
J_{prio} &= \sum_{i=0}^{N-1} \mathcal{E}_{comf,i/k}^T W_{comf} \mathcal{E}_{comf,i/k} + \mathcal{E}_{comf,i/k}^T W_{comf} \\
&\quad + \mathcal{E}_{x_n/k}^T W_{x_n} \mathcal{E}_{x_n/k} + \mathcal{E}_{x_n/k}^T W_{x_n}
\end{aligned}
\tag{17}
$$

where $\varepsilon_{comf,\,i/k}$ is the slack variable on the constraint enforcing driving comfort envelope (12) and $\varepsilon_{x_n/k}$ is the slack variable on the terminal set. $Q_f$ and $X_f^k$ are the terminal cost and set (See the next section).

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# Optimal Control Problem Formulation

## Weight Normalization and Prioritization

$$\begin{aligned} Q &= \text{diag}\left(\frac{q_{e_y}}{e_{y,max}}, \frac{q_{\dot{e}_y}}{\dot{e}_{y,max}}, \frac{q_{e_\psi}}{e_{\psi,max}}, \frac{q_{\dot{e}_\psi}}{\dot{e}_{\psi,max}}\right) \\ R_0 &= \frac{r_u}{u_{max}} \\ R_1 &= \frac{r_{\dot{u}}}{\dot{u}_{max}} \\ R_2 &= \frac{r_{\ddot{u}}}{\ddot{u}_{max}} \end{aligned} \tag{18}$$

**Setting priority:**

$$\begin{aligned} W_{x_n} &= w_{x_n} \times \text{norm}(Q, Inf) \\ W_{comf} &= w_{comf} \times \text{norm}(Q, Inf) \\ W_{x_n} &\gg W_{comf} \gg \text{norm}(Q, Inf) \end{aligned} \tag{19}$$

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# MPC stabilization using set invariance: Stability related “terminal ingredients”

## Computation of the Terminal Constraint[1]

Let $\sum = \{\rho \in \mathbb{R},\ \rho_{min} \ll \rho \ll \ \rho_{max}\}$ be the parameter space that describes all possible matrices $(A_d(\rho), B_d(\rho))$.

For a specific $\rho \ \in \sum$, a common selection for $X_f$ is the maximal control invariant set $C_\infty$ of the closed-loop system:

$$x(k+1) = (A_d(\rho(k)) + B_d(\rho(k))K_{LQR})x(k) \qquad (20)$$

where $K_{LQR}$ is the Linear quadratic regulator (LQR) gain.

The prediction model (10) is LPV, there are several maximal control invariant sets $C_\infty(\rho)$ and several different LQR feedback gains $K_{LQR}(\rho)$, each corresponding to a distinct $\rho \ \in \sum$. Therefore, the maximal control invariant set $C_\infty$, which remains invariant for all $\rho \ \in \sum$, is computed using the following recursion

$$\mathcal{O}_{k+1} = \bigcap_{\rho \in \Sigma_d} \mathrm{Pre}_\rho(\mathcal{O}_k) \cap \mathcal{O}_k, \ \ \mathcal{O}_0 = \mathbb{X} \qquad (21)$$

Ultimately, the recursion converges to

$$\mathcal{X}_f = \mathcal{C}_\infty = \lim_{k \to \infty} \mathcal{O}_k \qquad (22)$$

The notation $Pre_\rho(\cdot)$ denotes the set of states that evolve into the target set $(\cdot)$ in one time step.

[1] Pereira, G.C., et al. Linear Time-Varying Robust Model Predictive Control for Discrete-Time Nonlinear Systems. In 2018 IEEE Conference on Decision and Control. Miami Beach, FL, USA.

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# MPC stabilization using set invariance: Stability related “terminal ingredients”

## Computation of the Terminal Cost[1]

For a specific $\rho \in \Sigma$, a common selection for $Q_f$ is the infinite-horizon LQR cost $P(\rho)$.

$P(\rho)$ is the solution of the discrete-time algebraic Riccati equation for a specific $\rho \in \Sigma$.

For the LPV model, the terminal cost $Q_f$ is the unique $\bar{P}$ satisfying all the following linear matrix inequalities (LMIs):

$$A_{cl}(\rho)^T \bar{P}(\rho) A_{cl}(\rho) + K_{LQR}(\rho)^T R K_{LQR}(\rho) + Q - \bar{P}(\rho) \leq 0, \forall \rho \in \Sigma \quad (23)$$

with:

$$A_{cl}(\rho) = A_d(\rho(k)) + B_d(\rho(k)) K_{LQR} \quad (24)$$

**Remark 1:** The terminal stability cost and constraint are computed offline without accounting for input rate constraints and without the incorporation of slack variables.

**Remark 2:** To ensure that the QP is always feasible, slack variables are included in the state and input constraints and quadratically and linearly penalized in the cost function (16).

[1] Pereira, G.C., et al. Linear Time-Varying Robust Model Predictive Control for Discrete-Time Nonlinear Systems. In 2018 IEEE Conference on Decision and Control. Miami Beach, FL, USA.

## Highway A36 Exit to D1066 (N66) Scenario

Multi-scenario driving situation: succession of three different driving situations with speed defined by the driving rules respectively of 110km/h (highway), 50km/h (highway exit) and 90km/h (trunk road).

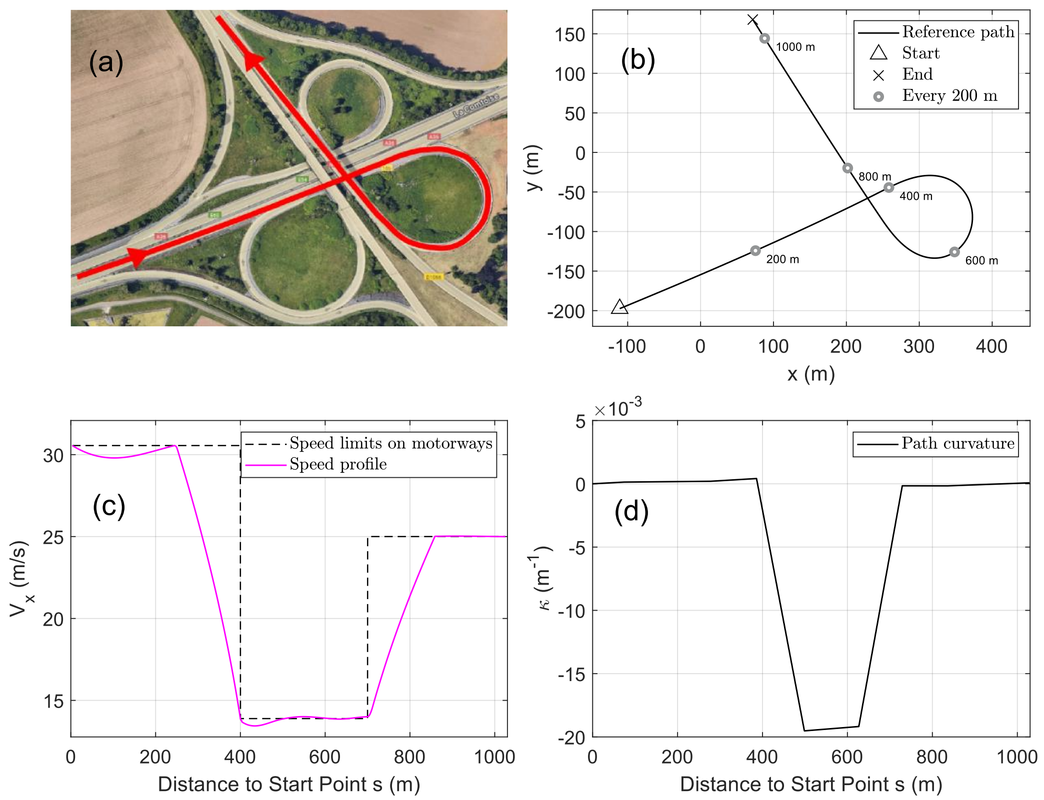


Fig. Road Description (a), Reference path (b), Speed profile (c), and Path curvature (d)

## Simulation Environment

- Controller prediction model: Vehicle Lateral dynamic model in terms of error with respect to road
- Controller validation vehicle model: IPG CarMaker high-fidelity model
- The LPV-MPC optimal control problem is transcribed into a QP problem (QP LPV-MPC)
- Uses qpOASES[1] as QP solver for the Optimization problem
- The controller runs at 50 Hz

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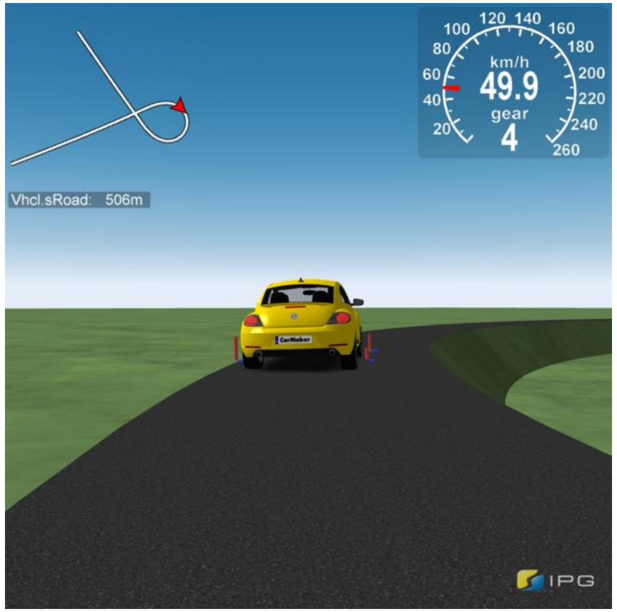



[1] Ferreau, H.J., Kirches, C., Potschka, A., Bock, H.G., and Diehl, M. (2014). qpOASES: a parametric active-set algorithm for quadratic programming. Mathematical Programming Computation, 6(4), 327-363.

# Simulation results

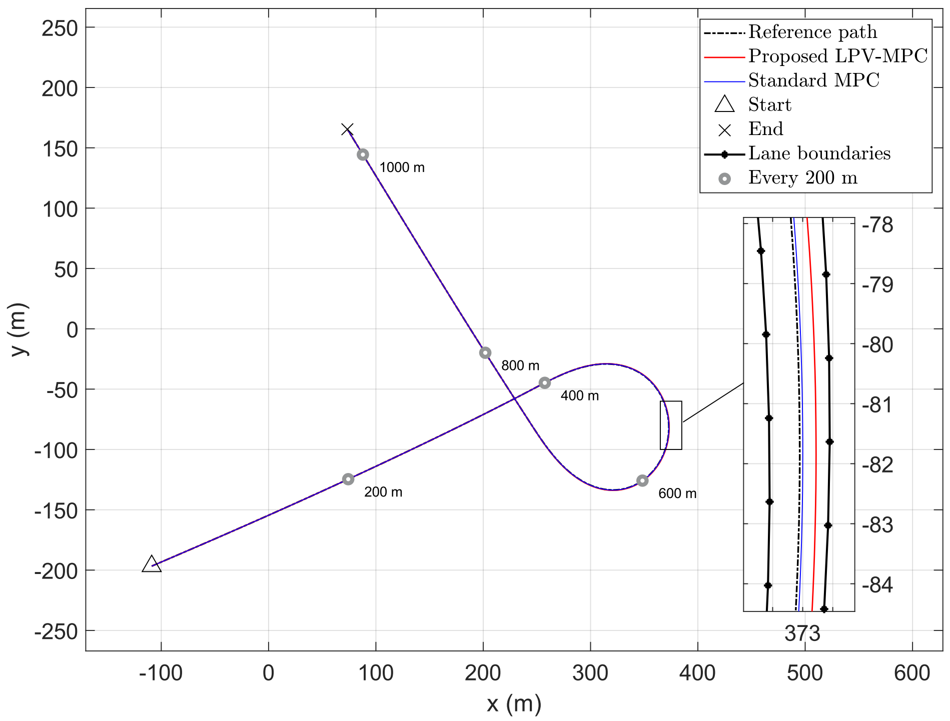



Fig. Vehicle Trajectory

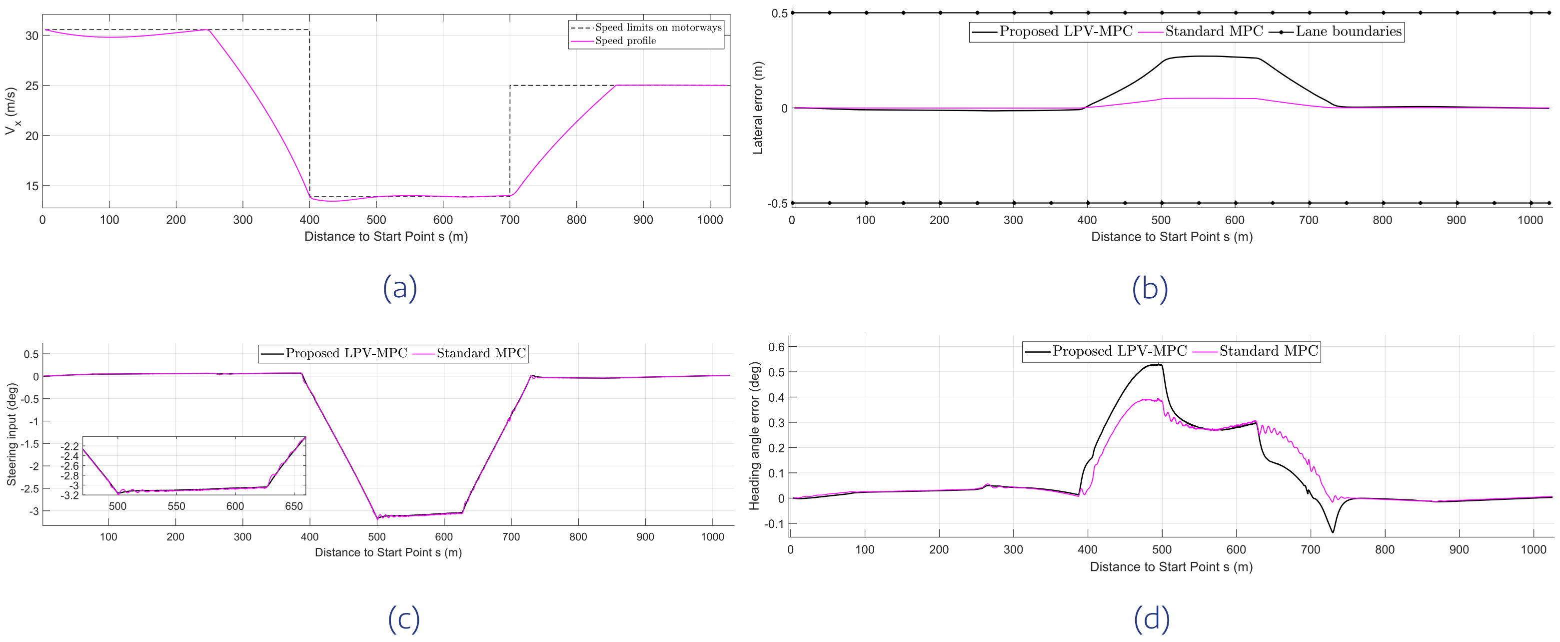



Fig. Speed profile (a), Lateral Position Error (b), Steering input (c), and Heading Angle Error (d)

# Performance assessment

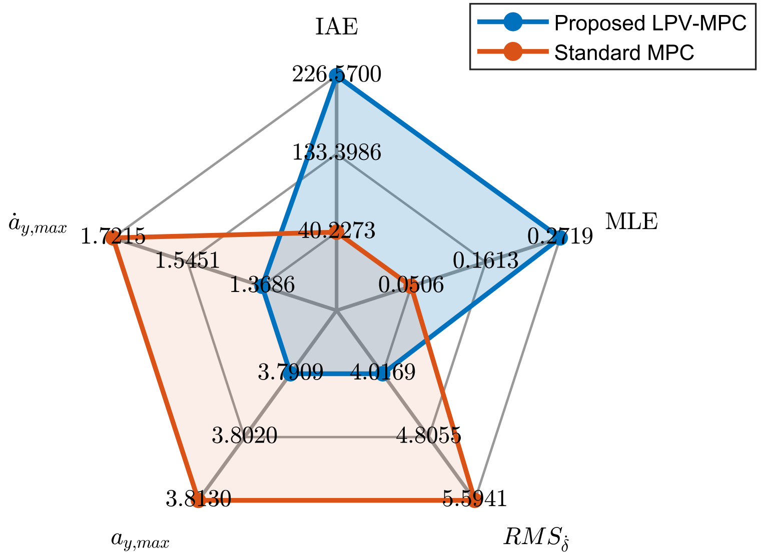
## A. Performance metrics (KPIs)

The following performance metrics are considered for performance evaluation:
Integral Absolute lateral Error ($IAE$), Maximum Lateral Error ($MLE$), Root mean square value of steering rate ($RMS_{\dot{\delta}}$), Lateral acceleration peak ($a_{y,max}$), Lateral jerk peak ($\dot{a}_{y,max}$)

| Definition | $IAE$ | $MLE$ | $RMS_{\dot{\delta}}$ | $a_{y,max}$ | $\dot{a}_{y,max}$ |
|---|---|---|---|---|---|
| Formulation | $\sum_{i=1}^{N}\lvert e_y\rvert$ | $max(\lvert e_y\rvert)$ | $\sqrt{\frac{1}{N}\sum_{i=1}^{N}\lvert\dot{\delta}\rvert^2}$ | $max(\lvert a_y\rvert)$ | $max(\lvert\dot{a}_y\rvert)$ |

## B. Performance results

| Controller Setup | Performance metrics | | | | |
|---|---|---|---|---|---|
| | IAE $[m]$ | MLE $[m]$ | $RMS_{\dot{\delta}}$ $[deg/s]$ | $a_{y,max}$ $[m/s^2]$ | $\dot{a}_{y,max}$ $[m/s^3]$ |
| Proposed LPV-MPC | 226.570 | 0.272 | 4.017 | 3.791 | 1.367 |
| Standard MPC | 40.227 | 0.051 | 5.594 | 3.813 | 1.722 |



## Summary

1. Proposal of an LPV-MPC control scheme that integrates path tracking, operational safety, and passengers' comfort
2. The proposed controller can be used to manage the trade-off between lane centering and passengers' comfort while guaranteeing safety in various driving scenarios
3. The restriction on steering input to account for passengers' comfort (See the equation below) tends to limit the steering input $\delta$ when the vehicle enters a big curvature turn, which could lead to insufficient steering with low tracking accuracy and a potential collision.

$$\delta_{max}(v_x) = \frac{l}{v_x^2} a_{y,base} + \mathcal{E}_{margin}$$

## Future developments

1. Guaranteed closed-loop stability of nonlinear model predictive controller[1]
   - (Adaptative) Control Lyapunov Function
   - Hamilton-Jacobi Reachability
2. Safety-critical environments[2]
   - Control Barrier Function: to verify and to enforce safety properties in the context of (optimization-based) safety-critical controllers
3. Robust extensions of the proposed MPC method to handle model mismatch and parametric uncertainty[3]

Fig. Super Agile Owl

[1] Razvan C. R., et al. Nonlinear Model Predictive Control using Lyapunov Functions for Vehicle Lateral Dynamics
[2] Aaron D. Ames. Control Barrier Functions: Theory and Applications
[3] Saltık, M. B., et al (2018). An outlook on robust model predictive control algorithms: Reflections on performance and computational aspects. Journal of Process Control, 61, 77-102.

# Thank you for your attention!

**Joyce SUDI** – PhD Student

Stellantis – Laboratoire IRIMAS - UR 7499
12th IFAC Symposium on Intelligent Autonomous Vehicles (IAV 2025)
joyce.sudibinali@stellantis.com

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